MATH 211 Exam #2 Solutions
Given on March 10, 2009

1. A group of people are surveyed about their television viewing habits. The table below summarizes the number of hours per week they reported viewing.

| Hours per week | frequency | probability |
|---|---|---|
| Less than 4 | 9 | |
| 5-8 | 15 | |
| 9-12 | 17 | |
| 13-16 | 13 | |
| 17-20 | 9 | |
| More than 20 | 4 | |

Use this information to fill in the "probability" column with the probability that a person chosen at random from this group would watch that many hours of TV.

| Hours per week | frequency | probability |
|---|---|---|
| Less than 4 | 9 | 9/76=0.134 |
| 5-8 | 15 | 15/67=0.224 |
| 9-12 | 17 | 17/67=0.254 |
| 13-16 | 13 | 13/67=0.194 |
| 17-20 | 9 | 9/67=0.134 |
| More than 20 | 4 | 4/67=0.060 |
| Sum | 67 | |

(a) What is the probability hat a person chosen at random from this group would watch less than 9 hours of TV per week?

$$P(X < 9) = 0.134 + 0.224 = 0.358$$

(b) What is the probability hat a person chosen at random from this group would watch at least 13 hours of TV per week?

$$P(X \geq 13) = 0.194 + 0.134 + 0.060 = 0.388$$

(c) What is the probability hat a person chosen at random from this group would watch between 9 and 20 hours of TV per week?

$$P(9 \leq X \leq 20) = 0.254 + 0.194 + 0.134 = 0.582$$

2. Consider the set of people
$S = \{$Mary, Seth, Taylor, Jorge, Kim, Sharla, Jurgen, Charles, Beth$\}$, and the subsets

   - $F = \{$Mary, Taylor, Kim, Sharla, Beth$\}$
   - $M = \{$Seth, Jorge, Jurgen, Charles$\}$
   - $T = \{$Mary, Seth, Jorge, Kim, Jurgen$\}$
   - $D = \{$Mary, Seth, Taylor, Jorge, Kim, Sharla$\}$.

   Suppose one person is selected at random from the set $S$. By $P(F)$, we mean the probability that this randomly chosen person would be in the set $F$. then $S$ is a sample space and $F, M, T, D$ are events.

   (a) Are any two of the events $F, M, T, D$ mutually exclusive?

   $F$ and $M$ are mutually exclusive, since there is no person who is in both $F$ and $M$. $F \cap M = \emptyset$.

   (b) Describe the event $F \cup T$ (i.e. tell me who is in that set) and calculate the probability $P(F \cup T)$.

   $$F \cup T = \{\text{Mary, Taylor, Kim, Sharla, Beth, Seth, Jorge, Jurgen}\}$$

   That's everyone who is in either $F$ or $T$.

   $$P(F \cup T) = \frac{n(F \cup T)}{n(S)} = \frac{8}{9}$$

   (c) Describe the event $F \cap D$ and calculate the probability $P(F \cap D)$.

   $$F \cap D = \{\text{Mary, Taylor, Kim, Sharla}\}$$

   That's everyone who is in both $F$ and $D$.

   $$P(F \cap D) = \frac{n(F \cap D)}{n(S)} = \frac{4}{9}$$

   (d) Describe the event $D^C$ and calculate the probability $P(D^C)$.

   $$D^C = \{\text{ Jurgen, Charles, Beth}\}$$

   That's everyone in $S$ who is *not* in $D$.

   $$P(D^C) = \frac{n(D^C)}{n(S)} = \frac{3}{9} = \frac{1}{3}$$

(e) Describe the event $D^C \cup T$ and calculate the probability $P(D^C \cup T)$.

$$D^C \cup T = \{\text{Mary, Seth, Jorge, Kim, Jurgen, Charles, Beth}\}$$

That's everyone who is either not in $D$ or is in $T$.

$$P(D^C \cup T) = \frac{n(D^C \cup T)}{n(S)} = \frac{7}{9}$$

3. Prudence Stapleton College consists of three academic divisions, Liberal Arts, Business, and Education. The following table gives a summary of the student body regarding their division and class standing.

| Division/Class | Freshman | Sophomore | Junior | Senior | totals |
|---|---|---|---|---|---|
| Liberal Arts | 375 | 281 | 237 | 180 | 1073 |
| Business | 512 | 360 | 280 | 210 | 1362 |
| Education | 212 | 312 | 325 | 356 | 1205 |
| totals | 1099 | 953 | 842 | 746 | 3640 |

(a) Find the probability that a randomly chosen student from Prudence Stapleton College is a Sophomore. $P(So)$

953 of the 3640 students are sophomores, so

$$P(So) = \frac{n(So)}{n(S)} = \frac{953}{3640} \approx 0.262$$

(b) Find the probability that a randomly chosen student from Prudence Stapleton College is a Business major. $P(Bu)$

1362 of the 3640 students are Business majors, so

$$P(Bu) = \frac{n(Bu)}{n(S)} = \frac{1362}{3640} = \frac{681}{1820} \approx 0.374$$

(c) Find the probability that a randomly chosen student from Prudence Stapleton College is a Junior majoring in Education. $P(Ju \cap Ed)$
325 of the 3640 students are Juniors majoring in Education, so

$$P(Bu) = \frac{n(Ju \cap Ed)}{n(S)} = \frac{325}{3640} = \frac{5}{56} \approx 0.0893$$

(d) Find the probability that a randomly chosen student from Prudence Stapleton College is either a Senior or is a Liberal arts major. $P(Se \cup LA)$

Since $Se$ and $LA$ are not mutually exclusive, we use the general addition rule,

$$P(Se \cup LA) = P(Se) + P(LA) - P(Se \cap LA) = \frac{746}{3640} + \frac{1073}{3640} - \frac{180}{3640} = \frac{1639}{3640} \approx 0.450$$

(e) Find the probability that a randomly chosen Freshman from Prudence Stapleton College is a Liberal Arts major. $P(LA|Fr)$

375 of the 1099 freshmen are Liberal Arts majors, so

$$P(LA|Fr) = \frac{n(LA \cap Fr)}{n(Fr)} = \frac{375}{1099} \approx 0.341$$

(f) Find the probability that a randomly chosen Education major from Prudence Stapleton College is a Senior. $P(Se|Ed)$

356 of the 1205 education majors are Seniors, so

$$P(Se|Ed) = \frac{n(Se \cap Ed)}{n(Ed)} = \frac{356}{1205} \approx 0.295$$

(g) Are being an Education major and being a Senior independent events? Justify your answer *mathematically.*

If being an Education major and being a Senior are independent events, it must be true that

$$P(Se|Ed) = P(Se) \qquad \text{and} \qquad \text{P(Ed|Se) = P(Ed)}$$

We need only check one of these conditions.

$$P(Se) = \frac{n(Se)}{n(S)} = \frac{746}{3640} \approx 0.205 \neq 0.295$$

In fact

$$\frac{P(Se|Ed)}{P(Se)} = 1.44$$

The proportion is not close to 1, therefore being a Senior and being an Education major at this college are not independent events.

4. A three county Arts Council consists of 7 members from Collier County, 12 members from Williams County, and 14 members from Ashton County. A three person committee is to be made up from the Council's membership. Suppose these committee members are chosen at random.

(a) What is the probability that the committee will have one member from each county?

$$P(CWA) = \frac{n(CWA)}{n(S)}$$

The denominator of this fraction is the number of different three member committees possible chosen from the 33 member Council. The numerator is the number of ways we could choose one member from Collier, one from Williams, and one from Ashton.

$$P(CWA) = \frac{n(CWA)}{n(S)} = \frac{7 \times 12 \times 14}{{}_{33}\mathrm{C}_3} \approx 0.216$$

(b) What is the probability that all three members will be from Ashton county?

The numerator is the number of ways we can choose three of the fourteen Ashton members.

$$P(AAA) = \frac{n(AAA)}{n(S)} = \frac{{}_{14}\mathrm{C}_3}{{}_{33}\mathrm{C}_3} \approx 0.0667$$

(c) What is the probability that all three members will be from the same county?

$$\begin{aligned} P(same) &= P(CCC) + P(WWW) + P(AAA) \\ &= \frac{{}_{7}\mathrm{C}_3}{{}_{33}\mathrm{C}_3} + \frac{{}_{12}\mathrm{C}_3}{{}_{33}\mathrm{C}_3} + \frac{{}_{14}\mathrm{C}_3}{{}_{33}\mathrm{C}_3} \\ &= 0.0064 + 0.0403 + 0.0667 \\ &= 0.113 \end{aligned}$$

(d) What is the probability that no one from Collier County is on the committee?

This would mean that all three members are chosen from the 26 members *not* from Collier.

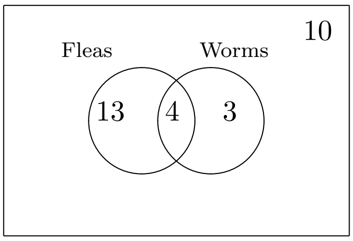
$$P(\text{no C}) = \frac{{}_{26}\mathrm{C}_3}{{}_{33}\mathrm{C}_3} = 0.477$$

(e) What is the probability that at least one county has no member on the committee?

Notice that this event is the complement of the event in part (a). Not "one member from each county" means at least one county with no member. We can use the complement rule.

$$P\left((CWA)^C\right) = 1 - P(CWA) = 1 - 0.216 = 0.784$$

5. A group of 30 dogs are inspected by veterinarians. 17 dogs are found to have fleas. 7 are found to have worms. 10 have neither fleas nor worms. (A Venn diagram may help.)




One way of completing the diagram above:

- there are 10 dogs with neither fleas nor worms. Since this region contains no boundaries, we can write that number in.
- There are 17 dogs with fleas and 10 dogs with neither fleas nor worms. That leaves 30-17-10=3 dogs with worms but not fleas.
- There are 7 dogs with worms. We just accounted for 3 of them, so that leaves 7-3=4 dogs with fleas and worms.
- There are 17 dogs with fleas. We just accounted for 4 of them. That leaves 17-4=13 dogs with fleas but not worms.

(a) If a dog is selected at random from this group, what is the probability that it has both fleas and worms?

From the diagram above, there are 4 dogs with both fleas and worms out of the 30 dogs, so

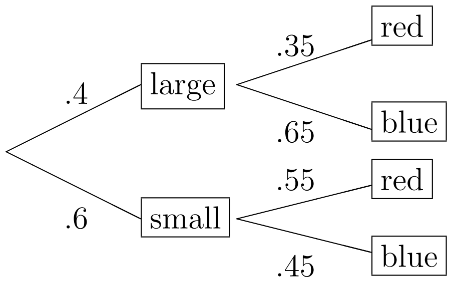
$$P(F \cap W) = \frac{4}{30} = \frac{2}{15}$$

(b) If a dog is inspected and found to have fleas, what is the probability that it also has worms?

There are 17 dogs with fleas, 4 of which also have worms, so

$$P(W|F) = \frac{4}{17}$$

6. I have a bag containing a large number of marbles. 40% of them are large $\frac{3}{4}$in diameter marbles, and the rest are $\frac{1}{2}$in diameter. 35% of the large marbles are red, and the rest of the large marbles are blue. 55% of the small marbles are red, and the rest of the small marbles are blue. (A tree diagram may help.)




$P(\text{large} \cap \text{red}) = .4 \times .35 = .140$

$P(\text{large} \cap \text{blue}) = .4 \times .65 = .260$

$P(\text{small} \cap \text{red}) = .6 \times .55 = ..330$

$P(\text{small} \cap \text{blue}) = .6 \times .45 = .270$

(a) If I reach in the bag and select a marble at random, what is the probability that it will be red?

$$\begin{aligned} P(\text{red}) &= P(\text{large} \cap \text{red}) + P(\text{small} \cap \text{red}) \\ &= .140 + .330 \\ &= .470 \end{aligned}$$

(b) If the marble I select is indeed red, what is the probability that it is large?

From the conditional probability rule,

$$P(\text{large}|\text{red}) = \frac{\text{P(large} \cap \text{red)}}{\text{P(red)}} = \frac{.140}{.470} = .299$$