MATH 211	Exam #2 Solutions
Given on	October 19, 2009

1. A group of 57 families was asked about their automobile ownership. The results are summarized in table 1

Table 1: Automobile Ownership

Number of autos	Frequency	Probability
0	3	$3/57 = .0526$
1	8	$8/57 = .140$
2	26	$26/57 = .456$
3	11	$11/57 = .193$
4	6	$6/57 = .105$
5	3	$3/57 = .0526$

(a) Fill in the "Probability" column of the table with the probability that a randomly chosen family in the survey would own that number of automobiles.

Divide the frequencies by 57.

(b) Find the probability that a randomly chosen family owns at least one automobile.

You can add the probabilities for 1,2,3,4 & 5 or use the rule of complements. The complement of "at least one automobile" is "no automobiles" so

$$P(\text{at least one}) = 1 - P(\text{none}) = 1 - .0526 = .947$$

(c) Find the probability that a randomly chosen family owns no more than three automobiles.

"no more than three automobiles" means 0, 1, 2 or 3 automobiles, so

$$\begin{aligned} P(\text{no more than three automobiles}) &= P(0) + P(1) + P(2) + P(3) \\ &= .0526 + .140 + .456 + .193 \\ &= .842 \end{aligned}$$

We can also calculate this by adding the corresponding frequencies and dividing by 57

$$P(\text{no more than two automobiles}) = \frac{3+8+26+11}{57} = .842$$

2. For the table in problem 1, we define the following events

   - A= the event that a randomly chosen family owns at least one automobile.
   - B= the event that a randomly chosen family owns no more than one automobile.
   - C= the event that a randomly chosen family owns one, two or three automobiles.
   - D= the event that a randomly chosen family owns more than two automobiles.

   (a) Is any pair of these events mutually exclusive? Which?

   A family which "owns no more than one automobile" (event B) cannot own "more than two automobiles" (event D). B and D are mutually exclusive. All others pairs of events have at least one family in common.

   (b) Describe the event $(B \cap C)$ and find its probability.

   In order to be in event $(B \cap C)$, a family must be included in both B and C. They must own one, two or three automobiles (C) but they cannot own more than one (B). Then they can only own one. So

   $$P(B \cap C) = P(1) = .140$$

   (c) Describe the event (D^C) and find its probability.

   The event (not D^C) includes all families who do *not* own more than two automobiles. Then they must own either zero, one, or two automobiles. So

   $$P(\text{not } D) = P(0) + P(1) + P(2) = .0526 + .140 + .456 = .649$$

(d) Describe the event $(B \cup C)$ and find its probability.

The event $(B \cup C)$ includes all families who own zero or one automobile (B) and also those who own one, two or three automobiles (C). Then $(B \cup C)$ includes all families who own zero, one, two or three automobiles. So

$$P(B \cup C) = P(0) + P(1) + P(2) + P(3) = .0526 + .140 + .456 + .193 = .842$$

3. A survey of 64 people concerning political affiliation is summarized in the following contingency table.

Table 2: Political Affiliations

Age	P_1 Republican	P_2 Democrat	P_3 Independent	Total
A_1: Under 30	4	5	1	10
A_2: 30-40	7	7	1	15
A_3: 40-50	6	8	0	14
A_4: 50-60	5	5	2	12
A_5: 60 and over	6	6	1	13
Total	28	31	5	64

Describe in words the following events and find their probabilities.

(a) (A_1)

The event (A_1) includes all those under 30. There are 10, and there are 64 people in the survey, so

$$P(A_1) = \frac{10}{64} = .156$$

(b) (P_2)

The event (P_2) includes the Democrats. There are 31, so

$$P(P_2) = \frac{31}{64} = .484$$

(c) $(A_2 \cap P_3)$

The event $(A_2 \cap P_3)$ includes those who are both 30-40 and Independent. There is only one Independent of age 30-40, so

$$P(A_2 \cap P_3) = \frac{1}{64} = .0156$$

(d) $(A_2 \cup P_3)$

The event $(A_2 \cup P_3)$ includes all those of age 30-40 *as well as* all Independents. We must take care not to count the same people twice. There are 5 Independents. Add to this the 14 people of age 30-40 who are not Independents (since we've counted the one who is). We get 19 people.

$$P(A_2 \cup P_3) = \frac{19}{64} = .297$$

Another method is to use the general addition rule.

$$P(A_2 \cup P_3) = P(A_2) + P(P_3) - P(A_2 \cap P_3) = \frac{15}{64} + \frac{5}{64} - \frac{1}{64} = .297$$

(e) Find the conditional probability P$(P_1|A_3)$

This is the probability that someone is a Republican, given that they are of age 40-50. There are 14 people of age 40-50. 6 of those are Republicans, so

$$P(P_1|A_3) = \frac{6}{14} = .429$$

Alternate method:

$$P(P_1|A_3) = \frac{P(P_1 \cap A_3)}{P(A_3)} = \frac{\frac{6}{64}}{\frac{14}{64}} = .429$$

(f) Find the conditional probability P$(A_3|P_1)$

This is the probability that someone is a of age 40-50, given that they are Republican. There are 28 Republicans. 6 of those are of age 40-50, so

$$P(A_3|P_1) = \frac{6}{28} = .214$$

Alternate method:

$$P(A_3|P_1) = \frac{P(A_3 \cap P_1)}{P(P_1)} = \frac{\frac{6}{64}}{\frac{28}{64}} = .214$$

4. If three people from the survey of problem #3 are chosen at random, find the following probabilities.

   (a) One is a Republican, one is a Democrat, and one is an Independent.

   There are ${}_{64}C_3 = 41664$ different sets of 3 people possible. We are choosing one of 28 Republicans, one of 31 Democrats, and one of 5 Independents.

$$P(RDI) = \frac{{}_{28}C_1 \cdot_{31} C_1 \cdot_5 C_1}{{}_{64}C_3} = \frac{28 \cdot 31 \cdot 5}{41664} = .104$$

   (b) Exactly two are Republicans.

   We must choose two of 28 Republicans and one non-Republican, of which there are 36.

$$P(2R) = \frac{{}_{28}C_2 \cdot_{36} C_1}{{}_{64}C_3} = .327$$

   (c) Exactly one is a Democrat.

   We choose one of 31 Democrats and two of 33 non-Democrats.

$$P(1D) = \frac{{}_{31}C_1 \cdot_{33} C_2}{{}_{64}C_3} = .393$$

(d) At least one is an Independent.

The simplest way to proceed is to use the rule of complements. Not "at least one" means none. There are 59 non-Independents, so

$$P(\text{at least one } I) = 1 - P(\text{no } I) = 1 - \frac{{}_{59}C_3}{{}_{66}C_3} = .220$$

5. A TV game show has 10 contestants. At the end of the show, a 1st, 2nd and 3rd place winner will be chosen from the ten. How many different outcomes are posible?

We are choosing three people from a list of ten, without replacement, and order is important. This is a permutation.

$${}_{10}P_3 = 720$$

There are 720 different outcomes possible.

6. In a group of 20 people, 7 own a dog, 9 own a cat, and 8 own neither a dog nor a cat. (Note: Some people may own both a dog and a cat.)

First, lets state the given information in terms of probabilities. The probability that a randomly selected person from this group owns a dog is $P(D) = \frac{7}{20}$. The probability that a randomly selected person from this group owns a cat is $P(C) = \frac{9}{20}$. The probability that a randomly selected person from this group owns neither a dog nor a cat is $P[\text{not } (D \cup C)] = \frac{8}{20} = \frac{2}{5}$.

(a) Find the probability that a randomly chosen member of this group owns *both* a dog and a cat.

We want $P(D \cap C)$. Note that

$$P(D \cup C) = 1 - P[\text{not } (D \cup C)] = 1 - \frac{2}{5} = \frac{3}{5}$$

We can use the general addition formula to find $P(D \cap C)$.

$$P(D \cup C) \quad = \quad P(D) + P(C) - P(D \cap C)$$

$$\begin{aligned}
\frac{3}{5} &= \frac{7}{20}+\frac{9}{20}-P(D\cap C)\\
\frac{3}{5} &= \frac{4}{5}-P(D\cap C)\\
P(D\cap C) &= \frac{4}{5}-\frac{3}{5}\\
P(D\cap C) &= \frac{1}{5}
\end{aligned}$$

(b) Find the probability that a randomly chosen dog owner also owns a cat.

$$P(C|D)=\frac{P(D\cap C)}{P(D)}=\frac{\frac{1}{5}}{\frac{7}{20}}=\frac{1}{5}\cdot\frac{20}{7}=\frac{4}{7}$$

There are other ways to approach this problem, e.g. Venn diagram or tree diagram.

7. Dave and his buddies get together for their regular Thursday night poker game, only to discover that Dave's dog has gotten a hold of their only deck of cards. Fortunately, the only cards that got chewed were face cards, so they decide to play without any face cards. That leaves only 40 cards, 1-10 in the four suits. Using this 40 card deck, calculate the probabilities of getting the following in five cards.

There are 40 cards in this deck, so there are ${}_{40}C_5$ different 5 card hands possible.

(a) Four tens.

There is only one set of four tens in the deck. In addition, we need one of the other 36 cards.

$$P(4\text{ tens})=\frac{1\cdot 36}{{}_{40}C_5}=.0000547$$

(b) Four of a kind (same rank).

The probability of getting four of any specified rank is the same as in part (a). Since there are ten ranks available, we multiply this by ten.

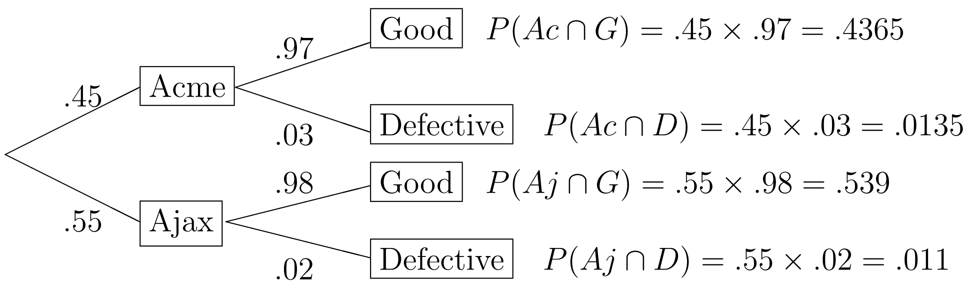
$$P(\text{4 of a kind}) = \frac{10 \cdot 36}{{}_{40}C_5} = .000547$$

(c) A full house (3 of one rank and 2 of another).

We think of this as choosing 3 of one rank and then two of another. There are 10 ranks available for the first three, and 9 ranks available for the pair. Hence

$$P(\text{Full House}) = \frac{10 \cdot {}_4C_3 \cdot 9 \cdot {}_4C_2}{{}_{40}C_5} = .00328$$

8. An electronics manufacturer buys light emitting diodes (LED's) from two companies, Acme and Ajax. They buy 45% of the LED's from Acme and the rest from Ajax. 3% of the Acme diodes are defective, while only 2% of the Ajax diodes are defective. (Suggestion: A tree diagram might be helpful here.)



(a) Find the probability that a randomly chosen LED is defective.

$$P(D) = P(Ac \cap D) + P(Aj \cap D) = .0135 + .011 = .0245$$

(b) A randomly chosen LED is found to be defective. Find the probability that it came from Acme.

$$P(Ac|D) = \frac{P(Ac \cap D)}{P(D)} = \frac{.0135}{.0245} = .551$$