Trigonometry	Exam 1B Solutions
Given on	Sept. 21, 2009

All nontrivial calculations must be shown.

1. For the equation $4x^2 + y^2 = 100$, find the x-intercepts and y-intercepts and test for symmetry about the x-axis, y-axis, and origin.

   - x-intercepts: Let $y = 0$ and solve for x.

$$\begin{aligned} 4x^2 + (0)^2 &= 100 \\ 4x^2/4 &= 100/4 \\ x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$

   - y-intercept: Let $x = 0$ and solve for y.

$$\begin{aligned} 4(0)^2 + y^2 &= 100 \\ y^2 &= 100 \\ y &= \pm 10 \end{aligned}$$

   - Symmetry about x-axis: Replace y with $-y$ and simplify.

$$\begin{aligned} 4x^2 + (-y)^2 &= 100 \\ 4x^2 + y^2 &= 100 \end{aligned}$$

     Since this simplifies to the original equation, its graph is symmetric with respect to the x-axis.

   - Symmetry about y-axis: Replace x with $-x$ and simplify.

$$\begin{aligned} 4(-x)^2 + y^2 &= 100 \\ 4x^2 + y^2 &= 100 \end{aligned}$$

     Since this simplifies to the original equation, its graph is symmetric with respect to the y-axis.

   - Symmetry about origin: We could check this by replacing x with $-x$ and y with $-y$, but it is not really necessary. It is not possible for a graph to exhibit exactly two of the three types of symmetry, hence the graph must be symmetric about the origin.

2. State the center and radius of the circle

$$(x - 2)^2 + (y + 5)^2 = 25$$

Comparing this to the general form $(x-h)^2 + (y-k)^2 = R^2$, we have

$$h = 2 \qquad k = -5 \qquad \text{and} \qquad R = \sqrt{25} = 5$$

The circle has its center at $(2, -5)$ and has radius 5.

3. Find the domains of the functions.

   (a) $f(x) = \sqrt{5x-2}$

   Even roots of negative numbers are not Real numbers, hence we must insure that the radicand is not negative.

$$\begin{aligned} 5x - 2 &\geq 0 \\ 5x - 2 + 2 &\geq 0 + 2 \\ 5x &\geq 2 \\ 5x/5 &\geq 2/5 \\ x &\geq 2/5 \end{aligned}$$

   The domain is $\{x | x \geq \frac{2}{5}\}$.

   (b) $g(x) = \dfrac{x^2+9}{x^2+5x}$

   Division by zero is not defined, hence we must insure that the denominator is not zero. We set the denominator equal to zero and solve.

$$\begin{aligned} x^2 + 5x &= 0 \\ x(x+5) &= 0 \end{aligned}$$

$$x = 0 \quad \text{or} \quad \begin{aligned} x + 5 &= 0 \\ x + 5 - 5 &= 0 - 5 \\ x &= -5 \end{aligned}$$

   So 0 and -5 are numbers that *cannot* be included in the domain. The domain is $\{x | x \neq 0, x \neq -5\}$.

4. Use the graph of the function to find

   (a) The domain: $[-9, 2] \cup [3, 7]$

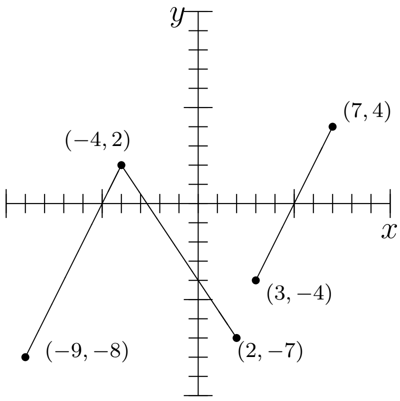
   (b) The range: $[-8, 4]$

   (c) y-intercept: $(0, -4)$

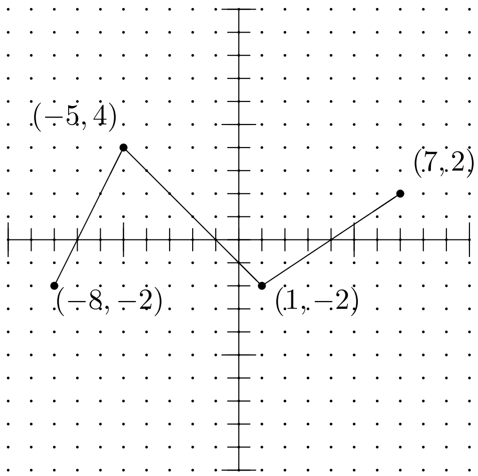
   (d) x-intercept(s): $(-5, 0), (-2.7, 0), (5, 0)$

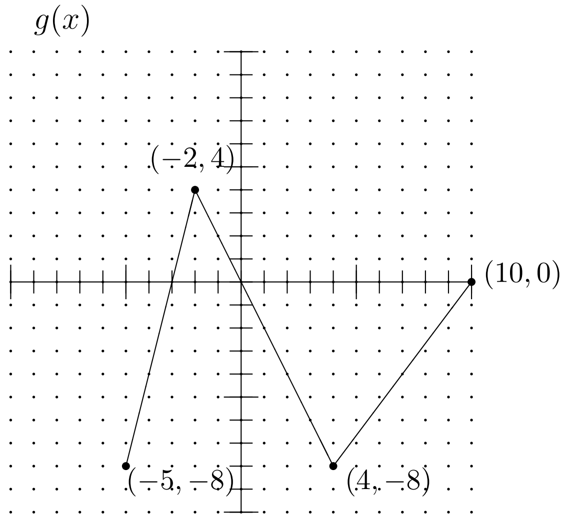
   (e) Increasing intervals: $(-9, -4) \cup (3, 7)$

   (f) Decreasing intervals: $(-4, 2)$



5. Shown below is a function $f(x)$. Graph the related function $g(x) = 2f(x-3) - 4$





The function $g(x)$ is $f(x)$ shifted to the right by 3 units, stretched along the y-axis by a factor of 2, and shifted down the y-axis by 4 units, in that order. We first determine where the labeled points will end up.

point	right by 3	y-stretch by 2	down by 4
$(-8,-2)$	$(-5,-2)$	$(-5,-4)$	$(-5,-8)$
$(-5,4)$	$(-2,4)$	$(-2,8)$	$(-2,4)$
$(1,-2)$	$(4,-2)$	$(4,-4)$	$(4,-8)$
$(7,2)$	$(10,2)$	$(10,4)$	$(10,0)$

Now we connect those points with line segments, and we have the transformed graph.

6. Convert the angles from degrees to radians.

(a) $240° = \underline{\frac{4\pi}{3}}$

$$240° \times \frac{\pi}{180°} = \frac{60 \times 4\pi}{60 \times 3} = \frac{4\pi}{3}$$

(b) $150^\circ = \underline{\ \frac{5\pi}{6}\ }$

$$150^\circ \times \frac{\pi}{180^\circ} = \frac{30 \times 5\pi}{30 \times 6} = \frac{5\pi}{6}$$

7. Convert the angles from radians to degrees.

(a) $\frac{4\pi}{3} = \underline{\ 240^\circ\ }$

$$\frac{4\pi}{3} \times \frac{180^\circ}{\pi} = \frac{4(3)(60^\circ)(\pi)}{3(\pi)} = 4(60^\circ) = 240^\circ$$

(b) $\frac{5\pi}{2} = \underline{\ 450^\circ\ }$

$$\frac{5\pi}{2} \times \frac{180^\circ}{\pi} = 5 \times 90^\circ = 450^\circ$$

8. Find the missing quantity.

(a) $r = 15''$ $\quad s = 22''$ $\quad \theta = \underline{\ \frac{22}{15}\text{rad}\ }$

$$\theta = \frac{s}{r} = \frac{22''}{15''} = \frac{22}{15}\text{rad}$$

(b) $\theta = 1.5$ $\quad r = 9$ meters $\quad s = \underline{\ 13.5 \text{ meters}\ }$

$$s = r\theta = 9(1.5) = 13.5 \text{ meters}$$

(c) $\theta = 35^\circ$ $\quad r = 8$ ft $\quad s = \underline{\ 4.89\text{ft}\ }$

$$\theta = 35^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{36}$$

$$s = r\theta = 8 \times \frac{7\pi}{36} = \frac{56\pi}{36} = \frac{14\pi}{9} \approx 4.89\text{ft}$$

Note that θ must be in radians

9. Express the angle $37^\circ 23' 31''$ in decimal degrees and then again in radians.

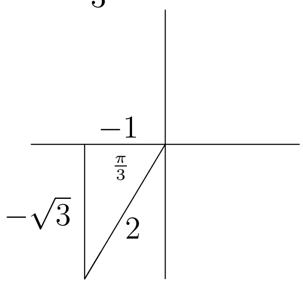
$$37^\circ 23' 31'' = 37^\circ + \frac{23^\circ}{60} + \frac{31^\circ}{3600} \approx 37.3919^\circ$$

In radians, this is

$$37.3919^\circ \times \frac{\pi}{180^\circ} = .6526\text{rad}$$

10. Find the *exact* values (not the calculator approximations) of the following trigonometric values.

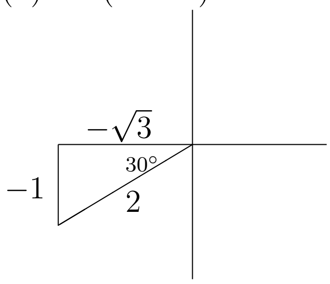
(a) $\cos\dfrac{4\pi}{3}$



$\dfrac{4\pi}{3} = \pi + \dfrac{\pi}{3}$, so the angle is in quadrant III and the reference angle is $\dfrac{\pi}{3}$. This is the standard $30^\circ - 60^\circ - 90^\circ$ triangle. In quadrant III, x and y are both negative. Then

$$\cos\frac{4\pi}{3} = \frac{\text{adj}}{\text{hyp}} = \frac{-1}{2} = -\frac{1}{2}$$

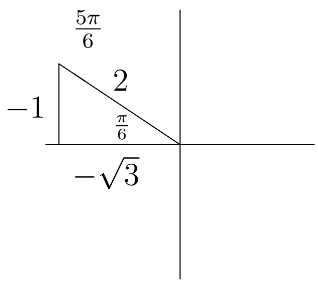
(b) $\sin(-150^\circ)$

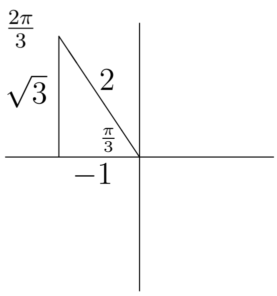


Negative angles are measure in the clockwise direction from the positive x-axis. $150^\circ = 180^\circ - 30^\circ$, so this angle is in quadrant III, and the reference angle is 30°. This is the standard $30^\circ - 60^\circ - 90^\circ$ triangle. Again in quadrant III, x and y are both negative. Then

$$\sin(-150^\circ) = \frac{\text{opp}}{\text{hyp}} = \frac{-1}{2} = -\frac{1}{2}$$

(c) $\cos\dfrac{5\pi}{6} - \cos\dfrac{2\pi}{3}$





$\dfrac{5\pi}{6} = \pi - \dfrac{\pi}{6}$, so $\dfrac{5\pi}{6}$ is in quadrant II, the reference angle is $\dfrac{\pi}{6}$ and

$$\cos\frac{5\pi}{6} = \frac{\text{adj}}{\text{hyp}} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$

$\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, so $\frac{2\pi}{3}$ is in quadrant II, the reference angle is $\frac{\pi}{3}$, and

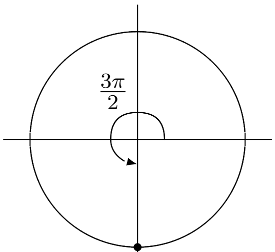
$$\cos\frac{2\pi}{3} = \frac{\text{adj}}{\text{hyp}} = \frac{-1}{2} = -\frac{1}{2}$$

Then

$$\cos\frac{5\pi}{6} - \cos\frac{2\pi}{3} = -\frac{\sqrt{3}}{2} - \left(-\frac{1}{2}\right) = \frac{1-\sqrt{3}}{2}$$

(d) $\sin\frac{7\pi}{2}$

$$\sin\frac{7\pi}{2} = \sin\frac{7\pi}{2} - 2\pi = \sin\frac{3\pi}{2} = -1$$



$(0,-1)$ Using the unit crcle definition, $\sin t = y$, so $\sin\frac{3\pi}{2} = -1$

11. Use a calculator to find approximations for the following trigonometric values.

(a) $\sin 50° =$ <u>0.766</u>

Make sure your calculator is in *degree* mode.

(b) $\csc\frac{3\pi}{5} =$ <u>1.0515</u>

Make sure your calculator is in *radian* mode.

$$\csc\frac{3\pi}{5} = \frac{1}{\sin\frac{3\pi}{5}} \approx 1.0515$$

(c) $\sec(-0.75) =$ <u>1.3667</u>

Make sure your calculator is in *radian* mode.

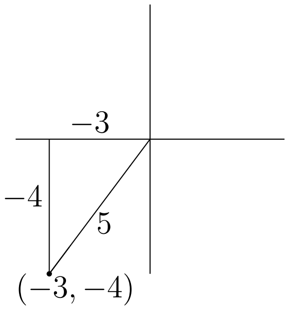
$$\sec(-0.75) = \frac{1}{\cos(-0.75)} \approx 1.3667$$

12. The point (-3,-4) is on the terminal side of an angle in standard position. Find the values of the trigonometric functions for this angle.

$\sin\theta = -\frac{4}{5}$ $\csc\theta = -\frac{5}{4}$

$\cos\theta = -\frac{3}{5}$ $\sec\theta = -\frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\cot\theta = \frac{3}{4}$



We can calculate the hypotenuse using the Pythagorean theorem.

$$\begin{aligned} h^2 &= (-3)^2 + (-4)^2 \\ &= 9 + 16 \\ &= 25 \\ h &= \sqrt{25} \\ &= 5 \end{aligned}$$