MATH 022 Exam #2B Solutions

Given on Oct 12, 2009

All work must be shown.

1. V is directly proportional to h and to the square of r. If $V = 50.265$ when $h = 4$ and $r = 2$, find V when $h = 3$ and $r = 3$.

$$\begin{aligned} V &= khr^2 \\ 50.265 &= k(4)(2)^2 \\ 50.265 &= 16k \\ 50.265/16 &= 16k/16 \\ k &\approx 3.142 \end{aligned}$$

So $V = 3.142hr^2$, and when $h = 3$ and $r = 3$,

$$V = 3.142(3)(3)^2 = 84.822$$

2. The illumination provided by a car's headlight varies inversely as the square of the distance from the headlight. A car's headlight provides an illumination of 3.75 footcandles at a distance of 40 feet. What is the illumination at a distance of 50 feet?

$$\begin{aligned} I &= \frac{k}{d^2} \\ 3.75 &= \frac{k}{40^2} \\ 3.75 \times 40^2 &= \frac{k}{40^2} \times 40^2 \\ k &= 6000 \end{aligned}$$

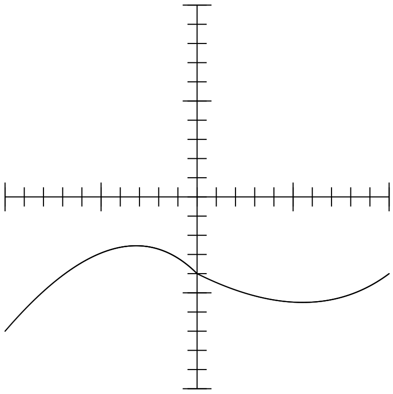
So $V = 6000/d^2$, and at a distance of 50 feet,

$$I = \frac{6000}{50^2} = 2.4 \text{ footcandles}$$

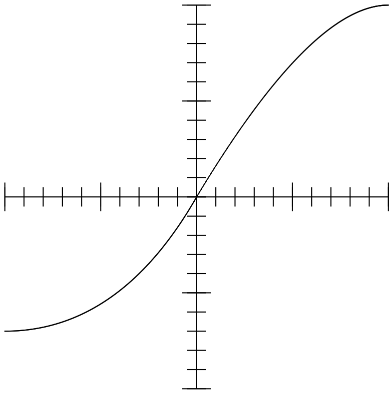
3. Which of the following defines a function? a,c,d,e,f

(a) $\{(1,3),(2,-5),(3,3),(4,7)\}$ (b) $\{(1,3),(2,-5),(3,8),(2,7)\}$

(c) $\{(1,3),(2,-5),(3,5),(4,7)\}$ (d) $\{(0,3),(2,-2),(4,3),(6,-7)\}$



(e)



(f)

4. Which of the items in problem #3 defines a one-to-one function? c,f

5. For the functions $f(x) = 3x - 5$ and $g(x) = x^2 + 4$, find the following:

(a) $f(3)$

$$f(3) = 3(3) - 5 = 9 - 5 = 4$$

(b) $g(-3)$

$$g(-3) = (-3)^2 + 4 = 9 + 4 = 13$$

(c) $f(2+a)$

$$f(2+a) = 3(2+a) - 5 = 6 + 3a - 5 = 1 + 3a$$

(d) $g(2a)$

$$g(2a) = (2a)^2 + 4 == 2^2a^2 + 4 = 4a^2 + 4$$

(e) $(f+g)(2)$

$$(f+g)(2) = f(2) + g(2) = [3(2) - 5] + \left[(2)^2 + 4\right] = 6 - 5 + 4 + 4 = 9$$

(f) $(g - f)(x)$

$$(g - f)(x) = g(x) - f(x) = x^2 + 4 - (3x - 5) = x^2 + 4 - 3x + 5 = x^2 - 3x + 9$$

(g) $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x) = (3x - 5)(x^2 + 4) = 3x^3 - 5x^2 + 12x - 20$$

(h) $(f/g)(0)$

$$(f/g)(0) = \frac{f(0)}{g(0)} = \frac{3(0) - 5}{(0)^2 + 4} = \frac{-5}{4} = -\frac{5}{4}$$

(i) $(f \circ g)(2)$

$$\begin{aligned}(f \circ g)(2) &= f\left(g(2)\right) \\ &= f\left((2)^2 + 4\right) \\ &= f(8) \\ &= 3(8) - 5 \\ &= 19\end{aligned}$$

(j) $(g \circ f)(x)$

$$\begin{aligned}(g \circ f)(x) &= g\left(f(x)\right) \\ &= g\left(3x - 5\right) \\ &= (3x - 5)^2 + 4 \\ &= (3x)^2 - 2(3x)(5) + (5)^2 + 4 \\ &= 9x^2 - 30x + 25 + 4 \\ &= 9x^2 - 30x + 29\end{aligned}$$

6. Verify that $f(x) = 4x - 5$ and $f^{-1}(x) = \dfrac{x + 5}{4}$ are inverse functions.

$$f\left(f^{-1}(x)\right) = f\left(\frac{x + 5}{4}\right) = 4\left(\frac{x + 5}{4}\right) - 5 = x + 5 - 5 = x$$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(4x - 5\right) = \frac{(4x - 5) + 5}{4} = \frac{4x}{4} = x$$

7. Find the inverse function of

(a) $f(x) = 4 - 2x$

$$\begin{aligned}
y &= 4 - 2x \\
y - 4 &= 4 - 2x - 4 \\
y - 4 &= -2x \\
\frac{y-4}{-2} &= \frac{-2x}{-2} \\
x &= -\frac{y-4}{2} = \frac{4-y}{2}
\end{aligned}$$

Interchanging x and y gives us

$$\begin{aligned}
y &= \frac{4-x}{2} \\
f^{-1}(x) &= \frac{4-x}{2}
\end{aligned}$$

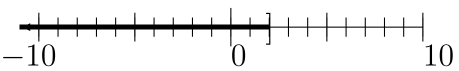
(b) $f(x) = \dfrac{3x+4}{x-1}$.

$$\begin{aligned}
y &= \frac{3x+4}{x-1} \\
y \times (x-1) &= \frac{3x+4}{x-1} \times (x-1) \\
yx - y &= 3x + 4 \\
yx - y - 3x &= 3x + 4 - 3x \\
yx - y - 3x &= 4 \\
yx - y - 3x + y &= 4 + y \\
yx - 3x &= 4 + y \\
x(y-3) &= y + 4 \\
\frac{x(y-3)}{y-3} &= \frac{y+4}{y-3} \\
x &= \frac{y+4}{y-3} \\
y &= \frac{x+4}{x-3} \\
f^{-1}(x) &= \frac{x+4}{x-3}
\end{aligned}$$

8. Solve the following inequalities or compound inequalities. Graph the solution on the number line.

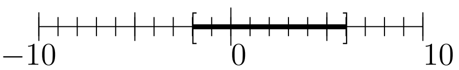
(a) $3(2x-5)+4x \leq 2(4x-3)-5$

$$\begin{aligned}
3(2x-5)+4x &\leq 2(4x-3)-5\\
6x-15+4x &\leq 8x-6-5\\
10x-15 &\leq 8x-11\\
10x-15-8x &\leq 8x-11-8x\\
2x-15 &\leq -11\\
2x-15+15 &\leq -11+15\\
2x &\leq 4\\
2x/2 &\leq 4/2\\
x &\leq 2
\end{aligned}$$



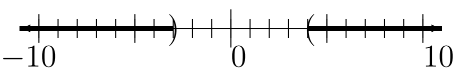
(b) $6x-5 \leq 12x+7$ and $3x-5 \leq x+7$

$$\begin{aligned}
6x-5 &\leq 12x+7 & &\text{and} & 3x-5 &\leq x+7\\
6x-5-6x &\leq 12x+7-6x & & & 3x-5-x &\leq x+7-x\\
-5 &\leq 6x+7 & & & 2x-5 &\leq 7\\
-5-7 &\leq 6x+7-7 & & & 2x-5+5 &\leq 7+5\\
-12 &\leq 6x & & & 2x &\leq 12\\
-12/6 &\leq 6x/6 & & & 2x/2 &\leq 12/2\\
-2 &\leq x & & & x &\leq 6\\
x &\geq -2 & &\text{and} & x &\leq 6
\end{aligned}$$



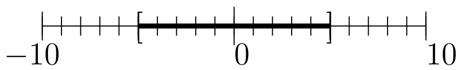
(c) $7x+16 < -5$ or $5x-12 > 2x$

$$\begin{aligned}
7x+16 &< -5 & &\text{or} & 5x-12 &> 2x\\
7x+16-16 &< -5-16 & & & 5x-12-2x &> 2x-2x\\
7x &< -21 & & & 3x-12 &> 0\\
7x/7 &< -21/7 & & & 3x-12+12 &> 0+12\\
x &< -3 & & & 3x &> 12\\
& & & & 3x/3 &> 12/3\\
x &< -3 & &\text{or} & x &> 4
\end{aligned}$$



(d) $-5 \le 5 - 2x \le 15$

$$\begin{array}{rcccl} -5 & \le & 5-2x & \le & 15 \\ -5-5 & \le & 5-2x-5 & \le & 15-5 \\ -10 & \le & -2x & \le & 10 \\ \frac{-10}{-2} & \ge & \frac{-2x}{-2} & \ge & \frac{10}{-2} \\ 5 & \ge & x & \ge & -5 \\ & & \text{or} & & \\ -5 & \le & x & \le & 5 \end{array}$$



9. Solve the equations.

(a) $|2x - 3| - 5 = 7$

$$\begin{aligned} |2x-3|-5 &= 7 \\ |2x-3|-5+5 &= 7+5 \\ |2x-3| &= 12 \end{aligned}$$

$$\begin{array}{rcl c rcl} 2x-3 &=& 12 & \text{or} & 2x-3 &=& -12 \\ 2x-3+3 &=& 12+3 & & 2x-3+3 &=& -12+3 \\ 2x &=& 15 & & 2x &=& -9 \\ 2x/2 &=& 15/2 & & 2x/2 &=& -9/2 \\ x &=& \frac{15}{2} & \text{or} & x &=& -\frac{9}{2} \end{array}$$

(b) $|3x - 5| = |2x + 3|$

$$\begin{array}{rcl c rcl} 3x-5 &=& 2x+3 & \text{or} & 3x-5 &=& -(2x+3) \\ 3x-5 &=& 2x+3 & & 3x-5 &=& -2x-3 \\ 3x-5-2x &=& 2x+3-2x & & 3x-5+2x &=& -2x-3+2x \\ x-5 &=& 3 & & 5x-5 &=& -3 \\ x-5+5 &=& 3+5 & & 5x-5+5 &=& -3+5 \\ x &=& 8 & & 5x &=& 2 \\ & & & & 5x/5 &=& 2/5 \\ x &=& 8 & \text{or} & x &=& \frac{2}{5} \end{array}$$

10. Solve the inequalities. State your solution in interval notation.

(a) $|5x-3| \geq 7$

The distnace between $5x-3$ and zero is at least 7. This implies that

$$\begin{array}{rclcrcl} 5x-3 & \leq & -7 & \text{0r} & 5x-3 & \geq & 7 \\ 5x-3+3 & \leq & -7+3 & & 5x-3+3 & \geq & 7+3 \\ 5x & \leq & -4 & & 5x & \geq & 10 \\ 5x/5 & \leq & -4/5 & & 5x/5 & \geq & 10/5 \\ x & \leq & -\frac{4}{5} & \text{or} & x & \geq & 2 \end{array}$$

In interval notation, this is $\left(-\infty, -\frac{4}{5}\right] \cup [2, \infty)$.

(b) $|3x-7| - 5 < 9$

$$\begin{array}{rcl} |3x-7|-5 & < & 9 \\ |3x-7|-5+5 & < & 9+5 \\ |3x-7| & < & 14 \end{array}$$

The distnace between $3x-7$ and zero is less than 14. This implies that

$$\begin{array}{rcccl} -14 & < & 3x-7 & < & 14 \\ -14+7 & < & 3x-7+7 & < & 14+7 \\ -7 & < & 3x & < & 21 \\ -7/3 & < & 3x/3 & < & 21/3 \\ -\frac{7}{3} & < & x & < & 7 \end{array}$$

In interval notation, this is $\left(-\frac{7}{3}, 7\right)$.

(c) $\left|\dfrac{4x-3}{5}\right| + 2 \leq 7$

$$\begin{array}{rcl} \left|\dfrac{4x-3}{5}\right| + 2 & \leq & 7 \\ \left|\dfrac{4x-3}{5}\right| + 2 - 2 & \leq & 7-2 \\ \left|\dfrac{4x-3}{5}\right| & \leq & 5 \end{array}$$

The distnace between $\left|\frac{4x-3}{5}\right|$ and zero is less than 5. This implies that

$$\begin{aligned}
-5 &\leq \frac{4x-3}{5} &\leq 5 \\
-5 \times 5 &\leq \frac{4x-3}{5} \times 5 &\leq 5 \times 5 \\
-25 &\leq 4x - 3 &\leq 25 \\
-25 + 3 &\leq 4x - 3 + 3 &\leq 25 + 3 \\
-22 &\leq 4x &\leq 28 \\
-22/4 &\leq 4x/4 &\leq 28/4 \\
-\frac{11}{2} &\leq x &\leq 7
\end{aligned}$$

In interval notation, this is $\left[-\frac{11}{2}, 7\right]$.