MATH 211 Exam #4 Solutions
Given on December 3, 2008

Unless otherwise indicated, assume populations are normally distributed.

1. The mean weight of adult Goode's Starlings is 200 grams with a standard deviation of 35 grams. Find the probability of choosing a random sample of 50 Goode's Starlings and getting the following:

   (a) a sample mean in excess of 210 grams.

   The standard deviation of the sample means for samples of size $n = 50$ is

   $$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{35}{\sqrt{50}} = 4.95$$

   We calculate the standardized z-score for 210.

   $$z = \frac{\overline{x} - \mu}{\sigma_{\overline{x}}} = \frac{210 - 200}{4.95} = 2.02$$

   Looking 2.02 on table IV, we find a probability of .4783. Since we want the area to the right of z, we must subtract this from .5.

   $$P(x > 210) = .5 - .4783 = .0217$$

   Using the TI-83 distribution function, you would enter normalcdf(210,500,200,4.95) to get the same result. Of course, any number at least 5 standard deviations above the mean would work in place of 500.

   (b) a sample mean of less than 195 grams

   $$z = \frac{\overline{x} - \mu}{\sigma_{\overline{x}}} = \frac{195 - 200}{4.95} = -1.01$$

   Looking up 1.01 in table IV, we find an area of .3438. Again, we want the area in the tail, so we must subtract this from .5.

   $$P(x < 195) = .5 - .3438 = .156$$

   The TI-83 distribution function is normalcdf(0,195,200,4.95), where any number less than $\mu - 5\sigma_{\overline{x}}$ could be used in place of 0.

(c) a sample mean between 195 grams and 210 grams

The simplest way to get this is to use what we have already done. Between 195 and 210 means not less that 195 nor greater than 210, so

$$\begin{aligned} P(195 < x < 210) &= 1 - P(x < 195) - P(x > 210) \\ &= 1 - .0217 - .156 \\ &= .822 \end{aligned}$$

You could confirm this with the TI-83 function normalcdf(195,210,200,4.95).

2. A standardized test of statistics skills was given to 150 randomly chosen Introductory Statistics students at various colleges across the country. The sample mean score was 88.3 with a sample standard deviation of 7.7. At the 95% confidence level, find a confidence interval for the mean if this test is given to all Introductory Statistics students.

$$\frac{\alpha}{2} = \frac{1 - .95}{2} = .025$$

To find $z_{\alpha/2} = z_{.025}$, we subtract

$$.5 - .025 = .475$$

We find this probability/area in table IV. The corresponding z-score is

$$z_{.025} = 1.96$$

The margin of error is

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{7.7}{\sqrt{150}} = 1.23$$

The confidence interval estimate for the mean score of all students is then

$$\overline{x} \pm E = 88.3 \pm 1.23 = (87.07, 89.53)$$

3. Below are lengths of a random selection of movies shown at local theaters. Find a 90% confidence level estimate for the average length of

all movies.

116	118	77	94	103	116	94
96	123	86	107	122	96	90
82	103	94	81	106	131	112
85	69	108	71	93	65	92
83	87	104	95	83	90	105

First, we calculate the mean and standard deviation. You can use the statistical features of your calculator or the formulas

$$\overline{x} = \frac{\Sigma x}{n} = 96.49 \qquad s = \sqrt{\frac{n\Sigma x^2 - (\Sigma x)^2}{n(n-1)}} = 15.92$$

$$\frac{\alpha}{2} = \frac{1 - .90}{2} = .05$$

To find $z_{.05}$, we look up $.5 - .05 = .45$ in the probabilities of table IV and find the corresponding z-value.

$$z_{.05} = 1.645 \qquad \text{(1.64 or 1.65 are acceptable)}$$

The margin of error is

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 1.645 \cdot \frac{15.92}{\sqrt{35}} = 4.43$$

We have used the sample standard deviation in place of the unknown population standard deviation. This is reasonable since $n = 35 > 30$. The confidence interval is then

$$\overline{x} \pm E = 96.49 \pm 4.43 = (92.06, 100.92)$$

4. The figures below are the weights in ounces of pepperoni on a sample of 20 of "Papa's" frozen pepperoni pizzas. At the 90% confidence level, find a confidence interval estimate for the mean weight of pepperoni on all "Papa's" frozen pepperoni pizzas.

4.09	4.37	4.10	3.91	4.35
3.68	3.63	4.59	4.37	3.69
4.44	4.41	4.24	4.15	4.82
3.60	3.95	3.97	3.99	3.96

First we calculate the mean and standard deviation.

$$\overline{x} = \frac{\Sigma x}{n} = 4.12 \qquad s = \sqrt{\frac{n\Sigma x^2 - (\Sigma x)^2}{n(n-1)}} = .334$$

Here $n = 20 < 30$, so we must use the students t-distribution. We have $n - 1 = 19$ degrres of freedom. The two-tailed value of α is $\alpha = 1 - .90 = .1$. Looking this up in table VI, we find $t_{.10} = 1.729$. The margin of error then is

$$E = t_\alpha \frac{s}{\sqrt{n}} = 1.729 \cdot \frac{.334}{\sqrt{20}} = .129$$

The confidence interval is then

$$\overline{x} \pm E = 4.12 \pm .129 = (3.99, 4.25)$$

5. Soda cans are filled at a factory by a machine. The standard deviation for the amount of soda put into cans is 0.25 ounce. At the 98% confidence level, how many cans should be checked in order to estimate the mean volume to within a margin of error of 0.1 ounce?

   First we find $z_{\alpha/2}$.

   $$\frac{\alpha}{2} - \frac{1 - .98}{2} = .01$$

   We find the probability $.5 - .01 = .49$ in table IV and find $z_{\alpha/2} = 2.33$. Then

   $$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{2.33 \times .25}{.1}\right)^2 = 33.94$$

   We would need to check 34 cans to estimate the mean to within 0.1 ounce.

6. Phoenixes have either green or red crests. In a randomly chosen sample of 60 phoenixes, 23 have green crests.

   (a) Find a 98% confidence interval estimate for the proportion of phoenixes with green crests.

We calculate $z_{\alpha/2}$ just as in the previous problem, and obtain $z_{\alpha/2} = 2.33$. we also need

$$\hat{p} = \frac{x}{n} = \frac{23}{60} = .3833 \qquad \hat{q} = 1 - \hat{p} = .6167$$

The margin of error is

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.33\sqrt{\frac{.3833 \times .6167}{60}} = .1462$$

and the confidence interval is

$$\hat{p} \pm E = .3833 \pm .1462 = (.237, .529)$$

(b) If we wish to have a margin of error no greater than 10%, how large a sample do we need?

$$n = \frac{(z_{\alpha}/2)^2\,(pq)}{(E)^2} = \frac{(2.33)^2(.3833 \times .6167)}{(.1)^2} = 128.3$$

We would need a sample of at least 129 phoenixes.