MATH 114 Exam #4B Solutions
Given on December 4, 2007

1. Perform the indicated operations.

   (a) i^{133}

   $133 \div 4 = 33$ with a remainder of 1. It is the remainder we need.

$$i^{133} = i^1 = i$$

   (b) $(7+3i) - (2-5i)$

$$(7+3i) - (2-5i) = 7 + 3i - 2 + 5i = (7-2) + (3+5)i = 5 + 8i$$

   (c) $(3-4i)(5+2i)$

$$\begin{aligned}(3-4i)(5+2i) &= 3 \times 5 + 3 \times 2i - 4i \times 5 - 4i \times 2i \\ &= 15 + 6i - 20i - 8i^2 \\ &= 15 - 14i - 8(-1) \\ &= 15 - 14i + 8 \\ &= 23 - 14i\end{aligned}$$

   (d) $\dfrac{3-i}{4-i}$

$$\begin{aligned}\frac{3-i}{4-i} &= \frac{3-i}{4-i} \cdot \frac{4+i}{4+i} \\ &= \frac{12 + 3i - 4i - i^2}{16 - i^2} \\ &= \frac{12 - i - (-1)}{16 - (-1)} \\ &= \frac{13 - i}{17} \\ &= \frac{13}{17} - \frac{1}{17}i\end{aligned}$$

2. Find all real and complex solutions to the following quadratic equations.

(a) $6x^2 - 6 = 5x$

$$\begin{aligned}
6x^2 - 6 - 5x &= 5x - 5x \\
6x^2 - 5x - 6 &= 0 \\
(3x + 2)(2x - 3) &= 0 \\
3x + 2 = 0 \quad &\text{or} \quad 2x - 3 = 0 \\
3x + 2 - 2 = 0 - 2 \quad &\text{or} \quad 2x - 3 + 3 = 0 + 3 \\
3x = -2 \quad & \quad 2x = 3 \\
\frac{3x}{3} = \frac{-2}{3} \quad & \quad \frac{2x}{2} = \frac{3}{2} \\
x = -\frac{2}{3} \quad & \quad x = \frac{3}{2}
\end{aligned}$$

(b) $4x^2 - 13 = 0$

$$\begin{aligned}
4x^2 - 13 + 13 &= 0 + 13 \\
4x^2 &= 13 \\
\frac{4x^2}{4} &= \frac{13}{4} \\
x^2 &= \frac{13}{4} \\
x &= \pm\sqrt{\frac{13}{4}} \\
x &= \pm\frac{\sqrt{13}}{\sqrt{4}} \\
x &= \pm\frac{\sqrt{13}}{2}
\end{aligned}$$

(c) $3t^2 - 4t - 2 = 0$

Snce the expression on the left does not factor, we use the quadratic formula, with $a = 3, b = -4, c = -2$.

$$\begin{aligned}
t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)} \\
&= \frac{4 \pm \sqrt{16 + 24}}{6} \\
&= \frac{4 \pm \sqrt{40}}{6} \\
&= \frac{4 \pm \sqrt{4}\sqrt{10}}{6}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4 \pm 2\sqrt{10}}{6} \\
&= \frac{2\left(2 \pm \sqrt{10}\right)}{6} \\
&= \frac{2 \pm \sqrt{10}}{3}
\end{aligned}
$$

(d) $x^2 + 5 = 4x$

$$
\begin{aligned}
x^2 + 5 - 4x &= 4x - 4x \\
x^2 - 4x + 5 &= 0
\end{aligned}
$$

Again, we use the quadratic formula, with $a = 1, b = -4, c = 5$.

$$
\begin{aligned}
x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} \\
&= \frac{4 \pm \sqrt{16 - 20}}{2} \\
&= \frac{4 \pm \sqrt{-4}}{2} \\
&= \frac{4 \pm \sqrt{4}\sqrt{-1}}{2} \\
&= \frac{4 \pm 2i}{2} \\
&= \frac{4}{2} \pm \frac{2i}{2} \\
&= 2 \pm i
\end{aligned}
$$

(e) $3(2x + 1)^2 - 5(2x + 1) + 2 = 0$

Make the substitution $u = 2x + 1$. Then the equation becomes

$$
\begin{aligned}
3u^2 - 5u + 2 &= 0 \\
(3u - 2)(u - 1) &= 0 \\
3u - 2 = 0 \quad &\text{or} \quad u - 1 = 0 \\
3u = 2 \quad & \quad u = 1 \\
\frac{3u}{3} = \frac{2}{3} \quad & \quad 2x + 1 = 1 \\
2x + 1 = \frac{2}{3} \quad & \quad 2x = 0 \\
2x = \frac{2}{3} - 1 \quad & \quad x = 0 \\
2x = -\frac{1}{3} \quad &
\end{aligned}
$$

$$\frac{1}{2} \cdot 2x = -\frac{1}{3} \cdot \frac{1}{2}$$
$$x = -\frac{1}{6}$$

(f) $y - 7\sqrt{y} + 12 = 0$

Let $u = \sqrt{x}$. Then the equation becomes

$$\begin{aligned} u^2 - 7u + 12 &= 0 \\ (u-4)(u-3) &= 0 \\ u - 4 = 0 \quad &\text{or} \quad u - 3 = 0 \\ u = 4 \quad & \quad u = 3 \\ \sqrt{y} = 4 \quad & \quad \sqrt{y} = 3 \\ (\sqrt{y})^2 = 4^2 \quad & \quad (\sqrt{y})^2 = 3^2 \\ y = 16 \quad & \quad y = 9 \end{aligned}$$

You should verify that these work in the original equation.

3. Find a polynomial equation which has solutions $x = 4, x = -2$, and $x = 0$.

$$\begin{aligned} x &= 4 \\ x - 4 &= 4 - 4 \\ x - 4 &= 0 \end{aligned} \qquad \begin{aligned} x &= -2 \\ x + 2 &= -2 + 2 \\ x + 2 &= 0 \end{aligned} \qquad \begin{aligned} x &= 0 \\ x &= 0 \\ x &= 0 \end{aligned}$$

The equation must have factors of $x-4$, $x+2$, and x. One such equation is

$$\begin{aligned} x(x-4)(x+2) &= 0 \\ x(x^2 - 2x - 8) &= 0 \\ x^3 - 2x^2 - 8x &= 0 \end{aligned}$$

4. For each of the following parabolas, find the x-intercepts, if they exist, the vertex, and sketch the graph.

(a) $y = -x^2 + 4x - 3$

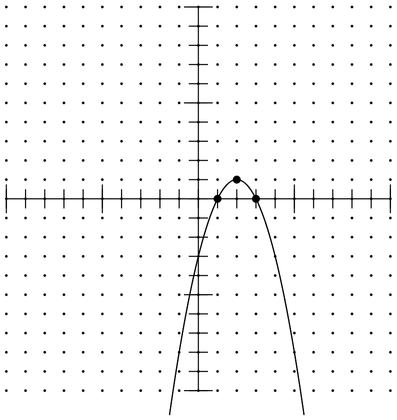
x-intercepts:

$$\begin{aligned} -x^2 + 4x - 3 &= 0 \\ -x^2(-1) + 4x(-1) - 3(-1) &= 0(-1) \\ x^2 - 4x + 3 &= 0 \\ (x-3)(x-1) &= 0 \\ x - 3 = 0 \quad &\text{or} \quad x - 1 = 0 \\ x = 3 \quad & \quad x = 1 \end{aligned}$$

vertex:

$$h = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$$

$$k = -(2)^2 + 4(2) - 3 = -4 + 8 - 3 = 1$$



vertex: $(2, 1)$ x-intercepts: $(1, 0), (3, 0)$

(b) $y = 2x^2 + 8x$

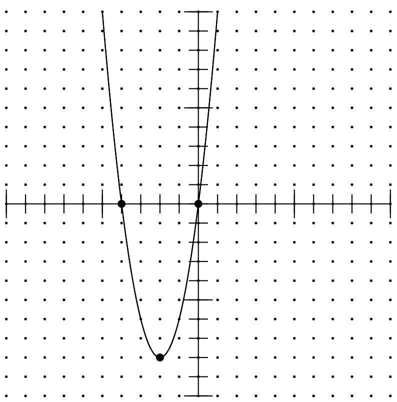
x-intercepts:

$$\begin{aligned} 2x^2 + 8x &= 0 \\ 2x(x+4) &= 0 \\ x = 0 \quad \text{or} \quad x + 4 &= 0 \\ x &= -4 \end{aligned}$$

vertex:

$$h = -\frac{b}{2a} = -\frac{8}{2(2)} = -2$$

$$k = 2(-2)^2 + 8(-2) = -8$$



vertex: $(-2, -8)$ x-intercepts: $(0, 0), (-4, 0)$

5. Solve the inequality and graph the solution set.

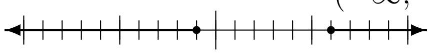
(a) $x^2 - 5x - 6 \geq 0$

$$\begin{aligned} x^2 - 5x - 6 &= 0 \\ (x-6)(x+1) &= 0 \\ x - 6 = 0 \quad &\text{or} \quad x + 1 = 0 \\ x = 6 \quad & \quad x = -1 \end{aligned}$$

	Test Pt	$(x-6)$	$(x+1)$	Result
$(-\infty, -1)$	-2	$-$	$-$	$+$
$(-1, 6)$	0	$-$	$+$	$-$
$(6, \infty)$	7	$+$	$+$	$+$

Since the inequality tells us that the expression should be greater than or equal to 0, we want the positive intervals, and the solutions to the equation are also solutions to the inequality. Hence, the solution set is

$$(-\infty, -1] \cup [6, \infty)$$



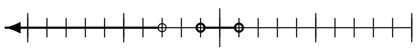
(b) $(x-1)(x+1)(x+3) < 0$

$$\begin{aligned} x - 1 &= 0 \\ x &= 1 \end{aligned} \qquad \begin{aligned} x + 1 &= 0 \\ x &= -1 \end{aligned} \qquad \begin{aligned} x + 3 &= 0 \\ x &= -3 \end{aligned}$$

	Test Pt	$(x-1)$	$(x+1)$	$(x+3)$	Result
$(-\infty, -3)$	-4	$-$	$-$	$-$	$-$
$(-3, -1)$	-2	$-$	$-$	$+$	$+$
$(-1, 1)$	0	$-$	$+$	$+$	$-$
$(1, \infty)$	2	$+$	$+$	$+$	$+$

Since the inequality tells us the expression should be strictly less than 0, we want the negative intervals. Our solution set is

$$(-\infty, -3) \cup (-1, 1)$$



(c) $\dfrac{x^2 - 9}{x + 1} < 0$

$$\begin{aligned} x^2 - 9 &= 0 \\ (x-3)(x+3) &= 0 \\ x - 3 = 0 \quad &\text{or} \quad x + 3 = 0 \\ x = 3 \quad & \quad x = -3 \end{aligned}$$

$$
\begin{aligned}
x + 1 &= 0 \\
x &= -1
\end{aligned}
$$

	Test Pt	$(x-3)$	$(x+1)$	$(x+3)$	Result
$(-\infty, -3)$	-4	$-$	$-$	$-$	$-$
$(-3, -1)$	-2	$-$	$-$	$+$	$+$
$(-1, 3)$	0	$-$	$+$	$+$	$-$
$(3, \infty)$	4	$+$	$+$	$+$	$+$

Since the inequality tells us the expression should be strictly less than 0, we want the negative intervals. Our solution set is

$$(-\infty, -3) \cup (-1, 3)$$