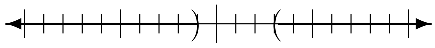
MATH 114 Exam # 2B Solutions
Given on October 9, 2007

1. Solve the following systems of inequalities and graph the solution set.

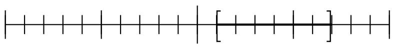
   (a) $5x - 7 > 8$ or $2x + 3 < 1$

$$\begin{array}{rclcrcl} 5x-7 & > & 8 & \text{or} & 2x+3 & < & 1 \\ 5x-7+7 & > & 8+7 & \text{or} & 2x+3-3 & < & 1-3 \\ 5x & > & 15 & & 2x & < & -2 \\ \frac{5x}{5} & > & \frac{15}{5} & & \frac{2x}{2} & < & \frac{-2}{2} \\ x & > & 3 & \text{or} & x & < & -1 \end{array}$$



   (b) $-5 \leq 9 - 2x \leq 7$

$$\begin{array}{rcccl} -5 & \leq & 9-2x & \leq & 7 \\ -5-9 & \leq & 9-2x-9 & \leq & 7-9 \\ -14 & \leq & -2x & \leq & -2 \\ \frac{-14}{-2} & \geq & \frac{-2x}{-2} & \geq & \frac{-2}{-2} \\ 7 & \geq & x & \geq & 1 \\ & & \text{or} & & \\ 1 & \leq & x & \leq & 7 \end{array}$$



2. Solve the following absolute value inequalities. Express your answer in interval notation.

   (a) $|4x - 3| \leq 9$

   The distance of $4x - 3$ from zero is less than 9, i.e.

$$\begin{array}{rcccl} -9 & \leq & 4x-3 & \leq & 9 \\ -9+3 & \leq & 4x-3+3 & \leq & 9+3 \\ -6 & \leq & 4x & \leq & 12 \\ \frac{-6}{4} & \leq & \frac{4x}{4} & \leq & \frac{12}{4} \\ -\frac{3}{2} & \leq & x & \leq & 3 \end{array}$$

   In interval notation, this is $\left[-\frac{3}{2}, 3\right]$.

(b) $|9-3x|-5>10$
First, isolate the absolute value.

$$\begin{aligned} |9-3x|-5 &> 10 \\ |9-3x|-5+5 &> 10+5 \\ |9-3x| &> 15 \end{aligned}$$

This means that $9-3x$ is more than 15 units from zero.

$$\begin{array}{rclcrcl} 9-3x &<& -15 & \text{or} & 9-3x &>& 15 \\ 9-3x-9 &<& -15-9 & & 9-3x-9 &>& 15-9 \\ -3x &<& -24 & & -3x &>& 6 \\ \frac{-3x}{-3} &>& \frac{-24}{-3} & & \frac{-3x}{-3} &<& \frac{6}{-3} \\ x &>& 8 & \text{or} & x &<& -2 \end{array}$$

In interval notation, this is $(-\infty,-2)\cup(8,\infty)$.

3. Factor the following expressions

(a) $4x^4-12x^3+9x^2$.

$$\begin{aligned} 4x^4-12x^3+9x^2 &= x^2(4x^2-12x+9) \\ &= x^2\left[(2x)^2-2(2x)(3)+3^2\right] \\ &= x^2(2x-3)^2 \end{aligned}$$

We are using the formula $a^2-2ab+b^2=(a-b)^2$ here.

(b) $27x^3+64$

$$\begin{aligned} 27x^3+64 &= (3x)^3+4^3 \\ &= (3x+4)\left[(3x)^2-(3x)(4)+4^2\right] \\ &= (3x+4)(9x^2-12x+16) \end{aligned}$$

Here we make use of the formula $a^3+b^3=(a+b)(a^2-ab+b^2)$.

(c) $x^3-3x^2-9x+27$

$$\begin{aligned} x^3-3x^2-9x+27 &= x^2(x-3)-9(x-3) \\ &= (x-3)(x^2-9) \\ &= (x-3)(x-3)(x+3) \\ &= (x-3)^2(x+3) \end{aligned}$$

(d) $24x^4-81x$

$$\begin{aligned} 24x^4-81x &= 3x(8x^3-27) \\ &= 3x\left[(2x)^3-3^3\right] \\ &= 3x(2x-3)\left[(2x)^2+(2x)(3)+3^2\right] \\ &= 3x(2x-3)(4x^2+6x+9) \end{aligned}$$

Here we make use of the formula $a^3-b^3=(a-b)(a^2+ab+b^2)$.

4. Solve the systems of equations on paper.

(a)

$$\begin{aligned} 3x + \ y &= \ 1 \\ x - 2y &= 12 \end{aligned}$$

Addition method: If we multiply the first equation by 2 and add it to the second, we will eliminate y.

$$\begin{aligned} 2 \times 3x + 2 \times y &= 2 \times 1 \\ 6x + 2y &= 2 \end{aligned}$$

$$\begin{array}{rl} 6x + 2y = & 2 \\ x - 2y = & 12 \\ \hline 7x \quad = & 14 \\ \frac{7x}{7} \quad = & \frac{14}{7} \\ x \quad = & 2 \end{array}$$

Using this value and the first equation, we dtermine y.

$$\begin{aligned} 3(2) + y &= 1 \\ 6 + y - 6 &= 1 - 6 \\ y &= -5 \end{aligned}$$

The solution is the point $(2, -5)$.

(b)

$$\begin{aligned} x - 2y &= 7 \\ y &= 2x - 8 \end{aligned}$$

Substitution method: We use $2x - 8$ in place of y in the first equation.

$$\begin{aligned} x - 2(2x - 8) &= 7 \\ x - 4x + 16 &= 7 \\ -3x + 16 - 16 &= 7 - 16 \\ -3x &= -9 \\ \frac{-3x}{-3} &= \frac{-9}{-3} \\ x &= 3 \end{aligned}$$

Using this value in the second equation, we obtain

$$y = 2(3) - 8 = -2$$

The solution is $(3, -2)$.

(c)

$$\begin{aligned} 2x + \ y - z &= \ 0 \\ x + 3y + z &= \ 2 \\ 3x - 2y + z &= 11 \end{aligned}$$

We can add the first two equations to eliminate z. We can also add the first and third equations and eliminate z.

$$\begin{array}{r} 2x + y - z = 0 \\ x + 3y + z = 2 \\ \hline 3x + 4y \quad = 2 \end{array}$$

$$\begin{array}{r} 2x + y - z = 0 \\ 3x - 2y + z = 11 \\ \hline 5x - y \quad = 11 \end{array}$$

This gives us two equations in the same two unknowns to solve. If we multiply the second of these by 4 and add the result to the first, we will eliminate y.

$$\begin{aligned} 4 \times 5x - 4 \times y &= 4 \times 11 \\ 20x - 4y &= 44 \end{aligned}$$

$$\begin{array}{r} 3x + 4y = 2 \\ 20x - 4y = 44 \\ \hline 23x \quad = 46 \\ \frac{23x}{23} \quad = \frac{46}{23} \\ x \quad = 2 \end{array}$$

We can use this value and the equation $5x - y = 11$ to determine y.

$$\begin{aligned} 5(2) - y &= 11 \\ 10 - y - 10 &= 11 - 10 \\ -y &= 1 \\ y &= -1 \end{aligned}$$

We can use these two values and the equation $x + 3y + z = 2$ to determine z.

$$\begin{aligned} (2) + 3(-1) + z &= 2 \\ -1 + z + 1 &= 2 + 1 \\ z &= 3 \end{aligned}$$

The solution is the point $(2, -1, 3)$.

5. Use the matrix features of your calculator to solve the following systems. Write down the augmented matrix you enter into your calculator to solve.

   (a)

$$\begin{aligned} 12x + 13y + 7z &= 7 \\ 13x + 7y + 12z &= -9 \\ 9x - 12y - 13z &= 32 \end{aligned}$$

The augmented coefficient matrix is

$$\begin{bmatrix} 12 & 13 & 7 & 7 \\ 13 & 7 & 12 & -9 \\ 9 & -12 & -13 & 32 \end{bmatrix}$$

If we put this into the TI-83 and use the rref([A]) function, we obtain the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 1.12056... \\ 0 & 1 & 0 & .81878... \\ 0 & 0 & 1 & -2.4415... \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \frac{1515}{1352} \\ 0 & 1 & 0 & \frac{1107}{1352} \\ 0 & 0 & 1 & -\frac{3301}{1352} \end{bmatrix}$$

We have used the MATH → Frac function to convert to fractions. So our solution is given by

$$x = \frac{1515}{1352} \qquad y = \frac{1107}{1352} \qquad z = -\frac{3301}{1352}$$

(b)

$$\begin{aligned} 7x \qquad +14z &= 57 \\ 3y - 5z &= -22 \\ 7x + 14y + 7z &= 8 \end{aligned}$$

The augmented coefficient matrix is

$$\begin{bmatrix} 7 & 0 & 14 & 57 \\ 0 & 3 & -5 & -22 \\ 7 & 14 & 7 & 8 \end{bmatrix}$$

If we put this into the TI-83 and use the rref([A]) function, we obtain the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 1.5714... \\ 0 & 1 & 0 & -1.8571... \\ 0 & 0 & 1 & 3.2857... \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \frac{11}{7} \\ 0 & 1 & 0 & -\frac{13}{7} \\ 0 & 0 & 1 & \frac{23}{7} \end{bmatrix}$$

The soluion is $x = \frac{11}{7} \qquad y = -\frac{13}{7} \qquad z = \frac{23}{7}$.

(c)

$$\begin{aligned} x + 3y + 2z &= 9 \\ 2x - 5y - z &= 5 \\ 4x + y + 3z &= 25 \end{aligned}$$

The augmented coefficient matrix is

$$\begin{bmatrix} 1 & 3 & 2 & 9 \\ 2 & -5 & -1 & 5 \\ 4 & 1 & 3 & 25 \end{bmatrix}$$

If we put this into the TI-83 and use the rref([A]) function, we obtain the matrix

$$\begin{bmatrix} 1 & 0 & .6363... & 0 \\ 0 & 1 & .4545... & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that the last row corresponds to the equation $0 = 1$. Since this equation can never be true, regardless of the values of x, y, z, there is no solution to the system. The system is inconsistent.

6. Find the slope of the line which contains the points $(3, -3)$ and $(-3, 5)$.

$$m = \frac{5 - (-3)}{-3 - 3} = \frac{8}{-6} = -\frac{4}{3}$$

7. Find the slope of the line $3x + 4y = 9$

The easiest way is to put the equation in slope-intercept form.

$$\begin{aligned}
3x + 4y &= 9 \\
3x + 4y - 3x &= 9 - 3x \\
4y &= -3x + 9 \\
\frac{4y}{4} &= \frac{-3x}{4} + \frac{9}{4} \\
y &= -\frac{3}{4}x + \frac{9}{4}
\end{aligned}$$

The slope is the coefficient of x. $\qquad m = -\frac{3}{4}$

8. Find equations of the the following lines.

(a) The line with slope $-\frac{4}{3}$ and y-intercept 3.

We can simply plug these values in to the slope-intercept form $y = mx + b$, to obtain

$$y = -\frac{4}{3}x + 3$$

(b) The line with slope -1 containing the point $(-2, 5)$. We can use the point-slope formula $y - y_1 = m(x - x_1)$.

$$\begin{aligned}
y - 5 &= -1(x - (-2)) \\
y - 5 &= -1(x + 2) \\
y - 5 &= -x - 2 \\
y - 5 + 5 &= -x - 2 + 5 \\
y &= -x + 3
\end{aligned}$$

(c) The line containing the points $(-1, 3)$ and $(5, 0)$.

First, we must calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{5 - (-1)} = \frac{-3}{6} = -\frac{1}{2}$$

Now we use this slope and one of the two given points. We can use the point-slope formula as in part (b), or we can use the slope-intercept form. Usint the point $(5, 0)$ and the slope-intercept form, we have.

$$\begin{aligned} y &= mx + b \\ y &= -\frac{1}{2}x + b \\ (0) &= -\frac{1}{2}(5) + b \\ 0 &= -\frac{5}{2} + b \\ 0 + \frac{5}{2} &= -\frac{5}{2} + b + \frac{5}{3} \\ b &= \frac{5}{2} \end{aligned}$$

Then the equation of the line is

$$y = -\frac{1}{2}x + \frac{5}{2}$$

(d) The line containing the points $(0, 5)$ and $(2, 5)$.

We calculate the slope.

$$m = \frac{5-5}{2-0} = \frac{0}{2} = 0$$

Since the slope is zero, the line is horizontal. The equation of a horizontal line is $y = b$, where b is the common y-coordinate of every point on the line (including the two given points.) Hence, the equation is

$$y = 5$$

(e) The line containing the point $(-3, -1)$ which is perpendicular to the line $2x + y = 15$.

First, we find the slope of the given line by putting it into slope-intercept form.

$$\begin{aligned} 2x + y &= 15 \\ 2x + y - 2x &= 15 - 2x \\ y &= -2x + 15 \end{aligned}$$

Then the slope is $m_2 = -2$. The line whose equation we want is perpendicular to this line, so it's slope must be

$$m_1 = -\frac{1}{m_2} = -\frac{1}{-2} = \frac{1}{2}$$

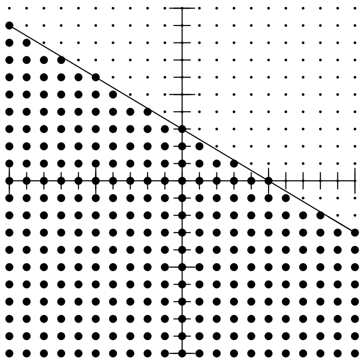
Now we have a slope and a point. Using the point-slope formula,

$$\begin{aligned} y - (-1) &= \frac{1}{2}(x - (-3)) \\ y + 1 &= \frac{1}{2}(x + 3) \\ y + 1 &= \frac{1}{2}x + \frac{3}{2} \end{aligned}$$

$$\begin{aligned} y + 1 - 1 &= \frac{1}{2}x + \frac{3}{2} - 1 \\ y &= \frac{1}{2}x + \frac{1}{2} \end{aligned}$$

9. Graph the linear inequalities.

   (a) $3x + 5y \leq 15$

   (b) $2x - 3 > 7$